Title: Bungee Barbie®

A Graphic Exploration of Harmonic Motion using CBL and Motion Detector

Link to Outcomes:

- **Communication** Students will describe and compare graphic and symbolic model both orally and in writing.
- **Reasoning** Students will collect data, analyze data, and make conjectures.
- **Cooperation** Students will demonstrate the ability to work in a team in the setup, execution, analysis, and summation of the experiment.
- **Trigonometry** Students will write equations of sinusoidal curves to fit data collected.
- **Connections** Students will recognize and demonstrate that trigonometric functions, graphic models, and physical models are interrelated.
- **Technology** Students will use the TI-82 graphics calculator, the CBL unit, and the motion detector probe.

Brief Overview:

Students will perform an experiment involving harmonic motion. They will then investigate the interrelationships among the physical phenomenon, the graphic model, and the symbolic representation.

Grade/Level:

Grades 11-12 Trigonometry/Pre-calculus/Physics

Duration:

This lesson is expected to take one to two class periods, depending on discussion and extensions used.

Prerequisite Knowledge:

Students must be familiar with writing equations of sinusoidal curves in standard form and with the use of the TI-82 graphics calculator.

Objectives:

- Analyze the amplitude, period, and vertical shift of a sinusoidal curve, and then write an equation.
- Become familiar with linking the TI-82, the CBL unit, and the motion probe.
- Use the equation to make predictions about time and distance of objects.
- Relate physical model, graphical model, and symbolic model.
- Interpret collected data, and apply to new situations.

Materials/Resources/Printed Materials:

- CBL unit
- TI-82 graphics calculator
- Vernier Motion Probe
- Calculator-to-calculator cable link
- Instructions for setting up experiment
- Instructions for Linking TI-82 Graphics Calculator
- Bungee cord or rubber band, 12 inches long
- Barbie®, Ken®, or <u>any other doll of approximately the same size</u>
- Ring stand and support rod OR
- Two chairs on table with meter stick between

THIS EXPERIMENT REQUIRES DOWNLOADING OR ENTERING THE CBL, MOTION, AND PLOTS PROGRAMS INTO YOUR TI-82 CALCULATOR.
See Attachment for instructions.

Development/Procedures:

- Give students a general idea of the experiment and what to expect.
- Establish groups of two to four students (Teacher could modify and teach as a demonstration lesson).
- Give students instructions for linking TI-82, CBL unit, and motion probe. SEE STUDENT INSTRUCTION SHEET
- Give students instructions for setting up a physical model of an oscillating object attached to a spring-like device. SEE STUDENT INSTRUCTION SHEET.
- Give students an activity sheet to complete after data has been collected.
- Have students share their results with the class either orally or in writing.

Evaluation:

The ability to link and use CBL and motion probe will be evaluated by observation. Activity sheet can be collected and evaluated. After completion of activity sheet, students should be prepared to give a written description and/or oral presentation addressing any discovered relationships between the physical, graphic, and symbolic models. Teachers may want to consider using self evaluation and/or peer evaluation.

Extension/Follow Up:

Students could take the coordinates of all the maximum points on the graph and put them in two lists in the statistics mode. Then they could find the equation of the exponential curve by doing an exponential regression. This equation could be pasted in for the amplitude in the original equation to make the wave oscillate and decrease in amplitude simultaneously.

Students could repeat the activities in the experiment, varying the length of the bungee cord, or making Barbie® heavier to explore the effects.

Students could choose velocity-time plots from the plot option menu and analyze the graph by itself and by comparing it with the position versus time plot.

Students could bring in articles about bungee jumping, especially about the possible side effects. Articles on bungee jumping:

“Go Jump Off a Bridge”, Sports Illustrated, June 5, 1989, p. 12.
“The Ultimate Leap of Faith: In Bungee Madness, Life Hangs by a Thread”, Time, April 23, 1990, p. 75.

Students could continue making up problems about “Daredevil Barbie®,” such as Barbie® skydiving or Barbie® on a pendulum swing.

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PROCEDURE FOR LINKING AND DOWNLOADING TI-82 GRAPHICS CALCULATORS

1. Connect the TI-82 Link cable to both calculators at the I/O located at the bottom edge of the calculator. Make sure the cable ends are FLUSH to the calculator. If the cable is not connected correctly, the sender screen will show the message "Error in Xmit" when the sender tries to transmit.

2. Press **<2nd> LINK** on both calculators.

   <u>Receiver Directions:</u>

   Press the right arrow key. **RECEIVE** will be highlighted. Press **ENTER**. The screen will show the message **"Waiting..."** Once the programs have been transmitted, the screen will display the message **"Receiving..."** and that will be followed by the message **"Done."**

   <u>Sender Directions:</u>

   Choose option **2:Select All -** Press **ENTER**. Press the down arrow until you reach **CBL** then press **ENTER** to select. Press the down arrow until you reach **MOTION**. Press **ENTER** to select. Press the down arrow until you reach **PLOTS** and press **ENTER** to select. Arrow to the right to select **TRANSMIT**. Press **ENTER** to send the program to the receiver.

3. Remove the TI-82 Link cable.

BUNGEE BARBIE®
STUDENT INSTRUCTION SHEET
FOR LAB SET UP

<u>Overview:</u>

By hanging Barbie® from a “bungee” cord, you will investigate and interpret the graphic model of her motion as she is pulled down and released.

<u>You will need:</u>

TI-82	Elastic cord
Calculator-to-calculator cable link	Barbie® doll
CBL unit	Ring Stand
Vernier motion detector	Support rod

CHECK TO MAKE SURE YOUR TI-82 HAS A PROGRAM NAMED CBL. IF NOT, SEE INSTRUCTOR.

<u>Experiment Set Up:</u>

1. Link the TI-82 to the CBL with the calculator-to-calculator cable link. Be sure to plug into I/O ports at the bottom of each unit very firmly.

2. Plug motion detector into SONIC port on left side of CBL.

3. Tie elastic cord around Barbie®’s ankles.

4. Attach other end of elastic cord to support rod (Support rod could be a meter stick supported by two chairs.).

5. Place motion detector on floor face up directly under Barbie®. Barbie®'s head should be at least 50 centimeters (about 18 inches) from motion detector.

6. Practice Barbie®’s bouncing motion by pulling down on her head and releasing. She should bounce at least three times before coming to rest. If not satisfied, make appropriate adjustments (add weights to Barbie®, adjust rod, etc.).

7. Use a meter stick to measure the distance from the motion detector to the crown of Barbie®’s head.

8. Turn on CBL and TI-82.

Conducting the Experiment:

	Do the Following and check off as completed:	What should happen:
_____	Press **PRGM**	Alphabetical list of programs should appear
_____	Arrow down to **CBL** program	Arrow will appear on left side
_____	Press **ENTER**	Program for CBL appears
_____	Press **ENTER**	CBL menu appears
_____	Highlight MOTION	
_____	Press **ENTER**	Motion menu appears
_____	Highlight MOTION	
_____	Press **ENTER**	This will reset CBL to - - - and **"Enter collection time in seconds" will appear on TI-82 screen**
_____	Press **10**	? 10 will appear on screen
_____	Press **ENTER**	**"HIT ENTER TO START MOTION GRAPH"** will appear on screen
_____	Pull Barbie® down by hair	CBL will now graph her motion
_____	Press **ENTER**	
_____	Release her	
_____	**Be patient and wait**	When CBL is finished collecting this motion data, a new menu will appear on screen.
_____	On new menu, highlight DISTANCE-TIME	Graph will appear

If you are satisfied with the graph, show it to your teacher and ask politely for an activity sheet. If you are not satisfied with your graph, restart your program on the calculator for the experiment by pressing **CLEAR, ENTER, ENTER, ENTER, 5, ENTER**

STUDENT ACTIVITY WORKSHEET

Barbie® and Ken® were driving to Ocean City to visit her Aunt for the weekend. Along the way, they spot a bungee crane in a field. Barbie® screams with excitement because she has always wanted to try bungee jumping. She climbs the tower with haste. Ken®, who has been studying Trigonometry, and just happens to have a CBL and a motion detector, decides to collect data about Barbie®'s "flight." On the platform, a young man with a rose tattoo and a bushy beard and a glint in his eye, nonchalantly helps Barbie® attach the elastic cord to her delicate ankles. Barbie®'s heart rate has already increased exponentially, as she anticipates her plunge into adventure. Barbie® looks over her toes past the edge of the platform. The blood is pumping even faster now. She jumps. She thinks she will have a feeling of a bird-- soaring, dipping and flying, but instead the ground was rushing up toward her at an alarming rate. Her eyes feel as if she is wearing those slinky eyeglasses with eyeballs that fall out and spring back. Just as she is sure that she will die, there is a tremendous upward jerk as she is pulled towards the sky, followed immediately by another heart stopping plunge. Finally, the motion stops. Barbie® is not at all sure that her heart is even beating. They untie the cord, and Ken® catches her in his arms. As soon as she catches her breath, he shows her his graph, and asks her the following questions:

1.____________________ What does the x-axis represent?

2.____________________ What unit of measurement is used on the x-axis?

3.____________________ Use the **TRACE** button and the arrow keys to trace along the function. Look only at the x-values. What is the domain of the function?

4.____________________ How long did the jump last?

5.____________________ How often did Ken®'s CBL collect data?

6.____________________ What does the y-axis represent?

7.____________________ What unit of measurement is used on the y-axis?

8.____________________ **TRACE** along the function again. This time look at the y-values. What is the range?

9.____________________ Why didn't Ken®'s graph touch the x-axis?

10.____________________ What kind of function is this?

11.____________________ Use the **TRACE** button. What are the coordinates of the minimum point on the graph?

12.____________________ After how many seconds did Barbie® reach this minimum point?

13.____________________ How far above the ground was she at this minimum point?

14.____________________ How close did Barbie® come to crashing?

15.____________________ Describe in your own words what was happening to Barbie® at the minimum point.

16.____________________ Use the **TRACE** button to find the coordinates of the maximum point on the graph.

17.____________________ After how many seconds did Barbie® reach this maximum point?

18.____________________ Describe in your own words what was happening to Barbie® at this point.

19.____________________ **TRACE** from this maximum point to the next maximum point. You have now traced one complete cycle of the graph. We are going to write an equation for this part of the graph. First, find out how far apart the x-coordinates of the two maximum points are by subtracting them. This will be the length of one complete cycle, and is called the PERIOD. The answer is in seconds.

20.____________________ In order to write an equation of a cosine curve, we need to know how many cycles the graph completes in 2π or 6.28 seconds. Your graph did one cycle in ???? seconds. (Look at number 19). Set up a proportion to find out how many cycles it will complete in 2π seconds.

1 cycle/?? seconds = x cycles / 2π seconds

21.____________________ How many cycles did Barbie® complete in approximately 6.28 seconds?

22.____________________ **TRACE** back over the curve until the cursor is again on the minimum point. What is the equation of the horizontal line that passes through this point?

23.____________________ What is the equation of the horizontal line that passes through your first maximum point?

24.____________________ The horizontal line that is half-way between the minimum and maximum point is the location around which the bouncing occurs. What is the equation of this axis?

25.____________________ How high off the ground was Barbie® before she was pulled down?

26.____________________ Is there any relationship between the answer to 24 and the answer to 25? What is it? Why?

27.____________________ How far apart vertically are the maximum and minimum points? (Subtract the y-coordinates to find this distance.)

28.____________________ On this part of the curve, Barbie® bounced from a minimum to a maximum. Half of this action occurred above the new horizontal axis and half occurred below. Divide the vertical distance in half to find out how much occurred above and how much occurred below. This is called the AMPLITUDE.

29.____________________ Write the equation of the curve. The equation should be:

$$y = A \cos B (x) + C$$

Where A is the amplitude, B is how many cycles the curve completes in 2π and C is the location of the axis around which the action occurs.

The equation is now ready to be entered into your calculator in order to check how accurate your calculations are. If you did a good job, the graph of the equation should fit over the graph of the recorded data.

Press **y =** and enter your equation:
Press **GRAPH**

30.____________________ Show your two graphs to your teacher and have him or her sign here.

NEWS FLASH…NEWS FLASH…NEWS FLASH…

Mayor Barry has just announced that he is going to allow bungee jumping off Key Bridge to help alleviate the city's financial crisis…

Being a wild and foolish daredevil at heart, you rush to the bridge to be among the first to "Jump for D. C." You jump off the bridge and fall 122 feet before rebounding. Consider the point at which you rebound to be your starting point in time. You then rebound to a distance just 46 feet below the bridge after three seconds.

31.____________________ What is the equation that represents your vertical position as a function of time?

32.____________________ Would you be plunging or rebounding between the seventh and eighth second?

33.____________________ How far below the bridge will you be when you finally stop bouncing around?

34. ____________________ How will they get you down?

35. Consider the following:

Colorado recently legalized bungee jumping from towers and hot air balloons, but declared bungee jumping from bridges is still illegal. The state government considers this to be too hazardous.[1]

The bulls in Pamplona, Spain run through the streets each summer along with a crowd of over 1000 people. This year, for the first time in fifteen years, one of the runners was gored to death.[2]

What do you think the role of the government should be in regulating dangerous sports? What sports, if any, should be regulated. Please support your answer.

[1] "Bungee Jumping Comes of Age," Time, April 15, 1991, p. 50.
[2] Washington Post. July 14, 1995.